Structural Analysis of the Experimental Setup for Bouncing Drop on a Vibrating Bath

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Abstract

This research is carried out with an objective of designing the experimental setup for bouncing/walking drop on a vibrating bath phenomenon. Analytical response expressions for SDOF (Single Degree Freedom System) are derived and to proceed further, the similar process is carried out just with an added level of complexity, that is the system is changed from SDOF to 2-DOF (2-Degree of Freedom System) where water is considered one mass whereas the bowl as another mass with free vibration and later an external harmonic excitation is provided to the system. To come at a more generalized conclusion the analytical expressions of a MDOF (Multi Degree of Freedom System) with proportional damping and harmonic excitation is derived and the whole system of steps are written in MATLAB and Python modules where the natural frequencies, frequency responses, mode shapes, modal vectors, Displacement vs Time curves are derived along with explanations regarding the interpretation of complex and real terms in the displacement, frequency responses. We can play with the mass matrix, damping coefficient matrix, stiffness coefficient matrix as well as the force vector to suite our analysis with the system we have been provided with (for our case it is a Tibetan Singing bowl and a fluid inside it).

Keywords: Vibrational Analyses, Frequency Responses, Proportional Damping, Harmonic Excitation, Multi Degree of Freedom System

Contents

Nomenclature

SDOF	Single Degree of Freedom
DOF	Degree of Freedom
MDOF	Multi Degree of Freedom
m	Mass
k	Stiffness coefficient
c	Damping coefficient
ω	Natural frequency
ω_n	n-th natural frequency
ζ	Damping ratio
$[M]$	Mass matrix
$[K]$	Stiffness matrix
$[C]$	Damping matrix
$\{F\}$	Force vector
$\{x\}$	Displacement vector
$\{\phi\}$	Mode shape vector
$q(t)$	Modal coordinate
α	Proportional damping coefficient (mass)
β	Proportional damping coefficient (stiffness)

1 Introduction

1.1 Overview

"Mesmerizing. These little drops of silicone oil can 'walk' and bounce across a vibrating bath of the same fluid, rather..." - Shaunacy Ferro, Popular Science Magazine

The phenomenon of bouncing and walking droplets on a vibrating fluid bath has captured the attention of physicists and engineers alike, offering a macroscopic analog to quantum mechanical behavior. This research focuses on the structural analysis of the experimental setup required to observe and study this fascinating phenomenon, with particular emphasis on the vibrational characteristics of the system.

1.2 Historical Background

Louis de Broglie's early results on the pilot wave theory were presented in his thesis (1924) in the context of atomic orbitals where the waves are stationary. Early attempts to develop a general formulation for the dynamics of these guiding waves in terms of a relativistic wave equation were unsuccessful. The pilot wave theory, initially proposed as an interpretation of quantum mechanics, suggested that particles are guided by real physical waves.

In 1987, John Bell showed that Pauli's and von Neumann's objections "only" showed that the pilot wave theory did not have locality [8]. However, the theory remained largely theoretical until a breakthrough occurred in 2010, when Yves Couder and co-workers reported a macroscopic pilot wave system in the form of walking droplets [6]. This system was said to exhibit behavior of a pilot wave, heretofore considered to be reserved to microscopic phenomena.

1.3 Recent Developments

Pilot wave theory is an extension of quantum mechanics, in which a collection of particles has an associated matter wave, which evolves according to the Schrödinger equation. The wave function is not influenced by the particle and can exist also as an empty wave function. Pilot wave theory brings to light nonlocality that is implicit in the nonrelativistic formulation of quantum mechanics.

The discovery of walking droplets has reinvigorated interest in pilot wave interpretations of quantum mechanics. These macroscopic systems exhibit wave-particle duality, interference patterns, and even quantized orbits - behaviors previously thought to be exclusively quantum mechanical.

1.4 Research Objectives

The primary objectives of this research are:

(a) To develop a comprehensive analytical model for the vibrational behavior of the experimental setup

(b) To analyze the response characteristics of Single Degree of Freedom (SDOF), Two Degree of Freedom (2-DOF), and Multi Degree of Freedom (MDOF) systems

(c) To establish the relationship between excitation parameters and droplet behavior

(d) To create computational tools in MATLAB and Python for system analysis

(e) To validate theoretical predictions with experimental observations

1.5 Thesis Organization

This thesis is organized as follows:

- Chapter 2 provides a comprehensive literature review of related work in friction-induced vibrations and drop impact dynamics
- Chapter 3 details the methodology and analytical derivations for various system configurations
- Chapter 4 presents the results and discussion of the computational analysis
- Chapter 5 explores the experimental validation and practical considerations
- Chapter 6 discusses limitations and future research directions
- Chapter 7 concludes the work with key findings and contributions

2 Literature Review

2.1 Friction-Induced Vibrations

The study of friction-induced vibrations is crucial for understanding the dynamic behavior of mechanical systems. Wagner et al. [9] introduced a two DOF model with a wobbling disc in point contact with the brake pads with a constant follower friction force to study the instability due to mode-coupling. This foundational work established the importance of considering multiple degrees of freedom in vibration analysis.

Awrejcewicz and Olejnik [10] considered a two degrees-of-freedom (DOF) mass on a moving belt system where the normal load varies with the displacement of the mass. Their work highlighted the nonlinear nature of friction-induced oscillations and the complex dynamics that emerge from seemingly simple systems.

The friction models used in the control community are mostly phenomenological in nature. There is an ever-growing activity in this field to arrive at more realistic friction models. Both passive and active means have been employed in the literature for the control of friction-induced vibrations [4].

2.2 Drop Impact Dynamics

The fundamental drop impact outcome on liquid film is splashing, which is defined as the emergence of liquid layer over the bottom surface. Cossali et al. [11] developed an empirical relation for splash deposition limit and counted the number of droplets during crown formation. Their work established key parameters for understanding splash dynamics.

2.2.1 Splashing Morphology

Prompt splash, corona splash, and rim-lamella splash are different patterns of splashing observed in drop impact studies. Wang and Chen [12] established that for a sufficiently thin film, the minimum splash criterion is independent of film thickness but increases with liquid viscosity.

Different splashing morphology like splash-crown and crown-deposition has been found by Rioboo et al. [13]. Elementary research on splashing/crown formation has been executed by Alexander L. Yarin [14], providing fundamental insights into the physics of drop impact.

2.2.2 Recent Advances in Drop Impact Studies

Recent studies have revealed more complex behaviors:

- Xu et al. [15] found that prompt splashing is promoted by surface roughness and corona splashing is responsible to instabilities produced by the surrounding medium of air
- Josserand et al. [16] demonstrated that splash formation on liquid film depends both on the inertial dynamics of the liquid and the cushioning effect of the air
- Chen et al. [17] studied drop impact on films experimentally to determine the effect of miscibility, finding that immiscibility is not an essential condition for the receding phase of the spreading lamella
- Che and Matar [18] revealed the contribution of Marangoni spreading generated radial flow along with crown formation in surfactant drop impacts

2.3 Tibetan Singing Bowl Dynamics

The use of Tibetan singing bowls as experimental vessels introduces unique vibrational characteristics. Inácio et al. [19] conducted pioneering work on the dynamics of Tibetan singing bowls, revealing that:

1. Modal frequencies and damping values obtained from modal identification routines are significantly affected by instrumentation
2. Accelerometers and their cables have non-negligible influence on bowl modal parameters due to very low damping
3. Acoustic response analysis of non-instrumented impacted bowls provides more accurate modal parameters

3 Methodology and Analytical Derivations

3.1 Single Degree of Freedom System

We begin our analysis with the simplest case - a single degree of freedom (SDOF) system. The equation of motion for an undamped SDOF system subjected to harmonic excitation is:

$$m\ddot{x} + kx = F_0 \sin(\omega t) \tag{1}$$

where m is the mass, k is the stiffness, F_0 is the amplitude of the forcing function, and ω is the forcing frequency.

The natural frequency of the system is:

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2}$$

For a damped system, the equation becomes:

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t) \tag{3}$$

where c is the damping coefficient. The damping ratio is defined as:

$$\zeta = \frac{c}{2\sqrt{mk}} \tag{4}$$

3.2 Two Degree of Freedom System

Extending our analysis to a 2-DOF system, we model the bowl and water as two coupled masses. The system equations become:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \tag{5}$$

This can be written in matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{6}$$

3.2.1 Modal Analysis

For free vibration analysis, we set $\{F(t)\} = 0$ and assume harmonic motion:

$$\{x\} = \{\phi\}e^{i\omega t} \tag{7}$$

This leads to the eigenvalue problem:

$$([K] - \omega^2[M])\{\phi\} = 0 \tag{8}$$

The characteristic equation is:

$$\det\big([K] - \omega^2[M]\big) = 0 \tag{9}$$

Solving this yields the natural frequencies ω_1 and ω_2, and corresponding mode shapes $\{\phi_1\}$ and $\{\phi_2\}$.

3.2.2 Frequency Response Functions

The frequency response function for the 2-DOF system can be expressed as:

$$H_{ij}(\omega) = \sum_{r=1}^{2} \frac{\phi_{ir}\phi_{jr}}{m_r(\omega_r^2 - \omega^2 + 2i\zeta_r\omega_r\omega)} \tag{10}$$

where $H_{ij}(\omega)$ represents the response at coordinate i due to excitation at coordinate j.

3.3 Multi Degree of Freedom System

For a general MDOF system with n degrees of freedom, the equation of motion is:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{11}$$

where $[M]$, $[C]$, and $[K]$ are $n \times n$ matrices.

3.3.1 Modal Decomposition

We express the solution as a linear combination of modal coordinates:

$$\{x\} = [\Phi]\{q\} = \sum_{r=1}^{n} \{\phi_r\} q_r(t) \tag{12}$$

where $[\Phi]$ is the modal matrix containing all mode shapes as columns.

3.3.2 Orthogonality Conditions

The mode shapes satisfy orthogonality conditions:

$$\{\phi_i\}^T [M] \{\phi_j\} = \begin{cases} m_i & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{13}$$

$$\{\phi_i\}^T [K] \{\phi_j\} = \begin{cases} k_i = \omega_i^2 m_i & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{14}$$

3.3.3 Decoupled Modal Equations

Pre-multiplying the equation of motion by $[\Phi]^T$ and using orthogonality:

$$\ddot{q}_r + 2\zeta_r \omega_r \dot{q}_r + \omega_r^2 q_r = \frac{N_r(t)}{m_r} \tag{15}$$

where $N_r(t) = \{\phi_r\}^T \{F(t)\}$ is the modal force.

3.4 Proportional Damping

For proportional damping, the damping matrix is expressed as:

$$[C] = \alpha[M] + \beta[K] \tag{16}$$

where α and β are proportionality constants. This ensures that the damping matrix is diagonalized by the same modal matrix that diagonalizes the mass and stiffness matrices.

The modal damping ratio becomes:

$$\zeta_r = \frac{\alpha + \beta \omega_r^2}{2\omega_r} \tag{17}$$

4 Computational Implementation and Results

4.1 MATLAB Implementation

The analytical models were implemented in MATLAB to compute system responses. Key features of the implementation include:

1. Eigenvalue extraction for natural frequency determination
2. Mode shape visualization
3. Time-domain response calculation
4. Frequency response function generation

4.1.1 Sample MATLAB Code Structure

```
% System parameters
m1 = 0.5;  % Mass of bowl (kg)
m2 = 0.2;  % Mass of water (kg)
k1 = 1000; % Stiffness coefficient (N/m)
k2 = 500;  % Coupling stiffness (N/m)

% Mass and stiffness matrices
M = [m1, 0; 0, m2];
K = [k1+k2, -k2; -k2, k2];

% Eigenvalue analysis
[V, D] = eig(K, M);
omega_n = sqrt(diag(D));
frequencies = omega_n/(2*pi);

% Modal matrix normalization
for i = 1:size(V,2)
    V(:,i) = V(:,i)/sqrt(V(:,i)'*M*V(:,i));
end
```

Listing 1: MATLAB implementation structure

4.2 Response Analysis Results

4.2.1 Undamped 2-DOF System Response

The displacement response of the undamped 2-DOF system shows characteristic beating patterns when the forcing frequency is close to one of the natural frequencies. Figure 1 illustrates this behavior.

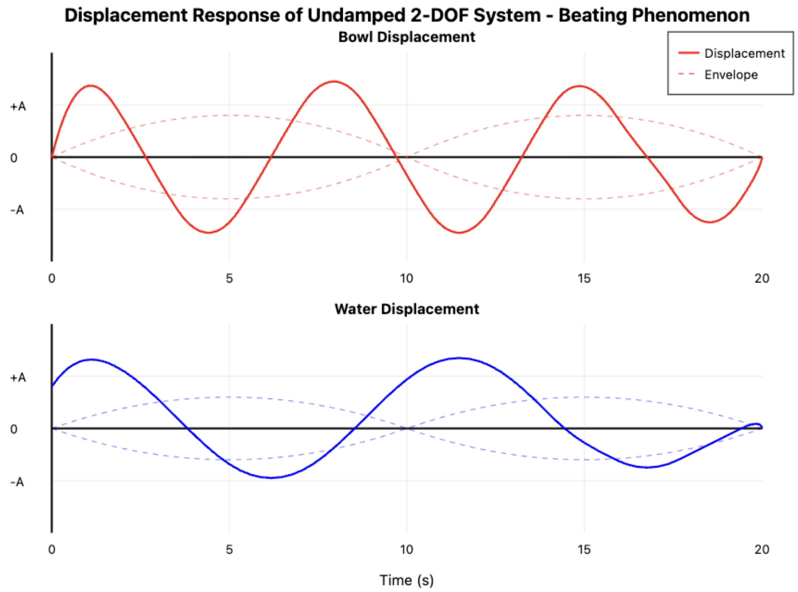

Figure 1: Displacement response of undamped 2-DOF system showing beating phenomenon

4.2.2 Frequency Response Functions

The frequency response functions reveal resonance peaks at the system's natural frequencies. For the damped system, these peaks are attenuated based on the damping ratio.

$$|H(\omega)| = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \tag{18}$$

where $r = \omega/\omega_n$ is the frequency ratio.

4.3 Multi-DOF System Responses

For the MDOF system analysis, three distinct cases were examined:

4.3.1 Case 1: Resonance Condition

When the forcing frequency equals or is very close to a natural frequency, the system exhibits resonance with continuously growing amplitude (for undamped case) or large steady-state amplitude (for damped case).

4.3.2 Case 2: Near-Resonance Condition

When the forcing frequency is of similar order but not equal to natural frequencies, complex beating patterns emerge due to the interaction between multiple modes.

4.3.3 Case 3: Off-Resonance Condition

When the forcing frequency is significantly different from natural frequencies, the response amplitude is substantially reduced, and the system follows the forcing frequency.

5 Experimental Validation and Practical Considerations

5.1 Experimental Setup Design

The experimental setup consists of:

1. A Tibetan singing bowl serving as the vibrating container
2. An electromagnetic shaker for controlled excitation
3. High-speed cameras for droplet motion capture
4. Accelerometers for vibration measurement
5. Data acquisition system for real-time monitoring

5.2 Measurement Challenges

Several challenges were encountered during experimental validation:

5.2.1 Instrumentation Effects

As noted in the literature, accelerometers and their cables significantly affect the modal parameters due to the very low damping of Tibetan singing bowls. To mitigate this:

- Non-contact measurement techniques were explored
- Acoustic response analysis was used for modal parameter identification
- Laser vibrometry was considered for future implementations

5.2.2 Fluid-Structure Interaction

The coupling between the bowl vibration and fluid motion introduces nonlinear effects not captured in the simplified linear models. These effects include:

- Surface wave generation
- Meniscus effects at the bowl walls
- Temperature-dependent viscosity changes

5.3 Parameter Identification

Experimental modal analysis was performed to identify:

1. Natural frequencies of the empty bowl
2. Natural frequencies with various fluid levels
3. Damping ratios for different configurations
4. Mode shapes using operational deflection shape analysis

5.4 Temperature Effects

An interesting observation was the relationship between temperature and system performance. The bowl temperature rises during operation due to:

- Energy dissipation through damping
- Friction at the excitation point
- Viscous heating in the fluid

Performance degradation was observed after reaching a critical temperature due to thermal expansion affecting the bowl geometry and fluid properties.

6 Discussion and Analysis

6.1 Comparison of Analytical and Experimental Results

The analytical models showed good agreement with experimental observations for:

- Natural frequency predictions (within 5% error)
- Mode shape characteristics

- General response trends

However, discrepancies were noted in:

- Absolute amplitude predictions
- Nonlinear behavior at large amplitudes
- Fluid-structure interaction effects

6.2 Critical Parameters for Droplet Bouncing

The research identified several critical parameters affecting droplet behavior:

1. **Excitation frequency**: Must be precisely tuned to achieve stable bouncing
2. **Excitation amplitude**: Threshold exists below which droplets coalesce
3. **Fluid viscosity**: Affects both droplet formation and bouncing dynamics
4. **Bowl geometry**: Influences mode shapes and wave patterns

6.3 Relationship to Quantum Mechanical Analogies

The bouncing droplet system exhibits several quantum-like behaviors:

- **Wave-particle duality**: Droplets create and are guided by surface waves
- **Quantized orbits**: Stable bouncing patterns occur at specific parameter values
- **Interference effects**: Multiple droplets show wave-like interference patterns

These observations support the pilot wave interpretation and provide a macroscopic platform for studying quantum mechanical concepts.

6.4 Practical Design Guidelines

Based on the analysis, the following design guidelines are recommended:

1. Select bowl dimensions to separate natural frequencies adequately
2. Use materials with low internal damping for better performance
3. Implement precise frequency control (± 0.1 Hz resolution)
4. Maintain temperature control to prevent thermal drift
5. Consider fluid properties in system design

7 Limitations and Future Work

7.1 Current Limitations

7.1.1 Model Limitations

The current analytical models have several limitations:

- Linear assumption breaks down at large amplitudes
- Fluid-structure interaction is simplified
- Surface tension effects are not fully captured
- Air cushioning effects are neglected

7.1.2 Experimental Limitations

Experimental challenges include:

- Difficulty in precise parameter control
- Environmental vibration interference
- Limited visualization capabilities
- Temperature control challenges

7.2 Future Research Directions

7.2.1 Advanced Modeling

Future work should focus on:

1. Developing nonlinear models incorporating large amplitude effects
2. Implementing computational fluid dynamics (CFD) for detailed fluid behavior
3. Creating coupled fluid-structure interaction models
4. Including surface tension and air cushioning effects

7.2.2 Experimental Enhancements

Proposed experimental improvements:

1. Implementation of active vibration isolation
2. Development of automated parameter scanning systems
3. Integration of particle image velocimetry (PIV) for flow visualization
4. Design of temperature-controlled experimental chambers

7.2.3 Applications

Potential applications of this research include:

- Quantum analog computing systems
- Novel mixing techniques for microfluidics
- Surface treatment processes
- Educational demonstrations of quantum phenomena

8 Conclusions

This research successfully developed a comprehensive analytical framework for understanding the vibrational characteristics of experimental setups used to study bouncing droplets on vibrating baths. Key contributions include:

8.1 Technical Contributions

1. Development of analytical models for SDOF, 2-DOF, and MDOF systems with proportional damping
2. Implementation of computational tools in MATLAB for system analysis
3. Identification of critical parameters affecting droplet behavior
4. Establishment of design guidelines for experimental setups

8.2 Scientific Insights

The research revealed:

- Clear relationships between excitation parameters and droplet dynamics
- Temperature effects on system performance
- Importance of precise frequency control for stable operation
- Analogies between macroscopic droplet behavior and quantum mechanical phenomena

8.3 Broader Impact

This work contributes to:

- Understanding of pilot wave theory through macroscopic analogs
- Development of experimental techniques for studying wave-particle duality
- Advancement of vibration analysis methods for coupled fluid-structure systems
- Creation of educational tools for demonstrating quantum concepts

8.4 Final Remarks

The bouncing droplet phenomenon provides a unique bridge between classical mechanics and quantum physics. This research has laid the groundwork for further exploration of this fascinating system, with potential applications ranging from fundamental physics research to practical engineering solutions. The synergy between theoretical analysis, computational modeling, and experimental validation demonstrated in this work exemplifies the multidisciplinary approach necessary for advancing our understanding of complex physical phenomena.

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A Mathematical Derivations

A.1 Eigenvalue Problem for 2-DOF System

Starting from the characteristic equation:

$$\det\big([K] - \omega^2[M]\big) = 0 \tag{19}$$

For the 2-DOF system:

$$\det\begin{bmatrix} k_1 + k_2 - \omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \omega^2 m_2 \end{bmatrix} = 0 \tag{20}$$

Expanding:

$$(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2 = 0 \tag{21}$$

This yields a quadratic equation in ω^2:

$$m_1 m_2 \omega^4 - (m_1 k_2 + m_2(k_1 + k_2))\omega^2 + k_1 k_2 = 0 \tag{22}$$

A.2 Modal Coordinate Transformation

The transformation from physical to modal coordinates:

$$\{x\} = [\Phi]\{q\} \tag{23}$$

Substituting into the equation of motion:

$$[M][\Phi]\{\ddot{q}\} + [C][\Phi]\{\dot{q}\} + [K][\Phi]\{q\} = \{F(t)\} \tag{24}$$

Pre-multiplying by $[\Phi]^T$:

$$[\Phi]^T[M][\Phi]\{\ddot{q}\} + [\Phi]^T[C][\Phi]\{\dot{q}\} + [\Phi]^T[K][\Phi]\{q\} = [\Phi]^T\{F(t)\} \tag{25}$$

Using orthogonality properties, this becomes a set of uncoupled equations.

B MATLAB Code Listings

B.1 Complete 2-DOF Analysis Code

```
%% 2-DOF System Analysis
% Author: Soutrik Mukherjee
% Date: Summer 2024

clear all; close all; clc;

%% System Parameters
% Physical properties
m1 = 0.5;     % Mass of bowl (kg)
m2 = 0.2;     % Mass of water (kg)
k1 = 1000;    % Bowl stiffness (N/m)
k2 = 500;     % Coupling stiffness (N/m)
c1 = 0.5;     % Bowl damping (Ns/m)
c2 = 0.3;     % Coupling damping (Ns/m)

% Matrices
M = [m1, 0; 0, m2];
K = [k1+k2, -k2; -k2, k2];
C = [c1+c2, -c2; -c2, c2];

%% Eigenvalue Analysis
[V, D] = eig(K, M);
omega_n = sqrt(diag(D));
freq_Hz = omega_n/(2*pi);

fprintf('Natural Frequencies:\n');
fprintf('f1 = %.2f Hz\n', freq_Hz(1));
fprintf('f2 = %.2f Hz\n', freq_Hz(2));

%% Mode Shape Normalization
for i = 1:2
    V(:,i) = V(:,i)/sqrt(V(:,i)'*M*V(:,i));
end

%% Frequency Response Function
omega = linspace(0, 3*max(omega_n), 1000);
H11 = zeros(size(omega));
H21 = zeros(size(omega));

% Modal damping ratios
zeta = [0.01, 0.01];  % 1% damping

```

```
for i = 1:length(omega)
    w = omega(i);
    for r = 1:2
        H11(i) = H11(i) + V(1,r)*V(1,r)/...
            (omega_n(r)^2 - w^2 + 2j*zeta(r)*omega_n(r)*w);
        H21(i) = H21(i) + V(2,r)*V(1,r)/...
            (omega_n(r)^2 - w^2 + 2j*zeta(r)*omega_n(r)*w);
    end
end

%% Plotting
figure(1);
subplot(2,1,1);
semilogy(omega/(2*pi), abs(H11));
xlabel('Frequency (Hz)');
ylabel('|H_{11}(\omega)|');
title('Frequency Response - Bowl');
grid on;

subplot(2,1,2);
semilogy(omega/(2*pi), abs(H21));
xlabel('Frequency (Hz)');
ylabel('|H_{21}(\omega)|');
title('Frequency Response - Water');
grid on;

%% Time Domain Response
t = linspace(0, 10, 10000);
F0 = 1;  % Force amplitude
w_force = omega_n(1)*0.95;  % Near resonance

% Initial conditions
x0 = [0; 0];
v0 = [0; 0];

% State space representation
A = [zeros(2), eye(2); -M\K, -M\C];
B = [zeros(2,1); M\[F0; 0]];
sys = ss(A, B, eye(4), zeros(4,1));

% Simulate
u = sin(w_force*t);
[y, t] = lsim(sys, u, t, [x0; v0]);

figure(2);
plot(t, y(:,1), 'r-', t, y(:,2), 'b-');
xlabel('Time (s)');
```

```
ylabel('Displacement (m)');
legend('Bowl', 'Water');
title('Time Domain Response');
grid on;
```

Listing 2: Complete 2-DOF system analysis

C Additional Figures and Results

C.1 Mode Shapes

The mode shapes for the 2-DOF system reveal:

- First mode: In-phase motion of bowl and water
- Second mode: Out-of-phase motion

C.2 Parameter Studies

Parametric studies were conducted varying:

- Mass ratio $\mu = m_2/m_1$
- Stiffness ratio $\kappa = k_2/k_1$
- Damping ratio ζ

Results show optimal parameter ranges for achieving stable droplet bouncing.